MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

EMT1016 – ENGINEERING MATHEMATICS I

(All Sections / Groups)

13 OCT 2017

9.00 A.M – 11.00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **4 pages** (including cover page) with **4** Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

(a) Consider the function $g(x) = 2 + \dfrac{1}{1+x}$.

(i) Sketch $g(x)$. Label all intersections at x-axis and y-axis.

[2 marks]

(ii) Perform continuity test for $g(x)$ at $x = 0$. Is $g(x)$ continuous at $x = 0$?

[3 marks]

(iii) At what x is $g(x)$ not continuous?

[1 mark]

(b) If $y = \dfrac{(x-2)^8 \cos(x)}{\sqrt[5]{2+5x}}$, use logarithmic differentiation to find $\dfrac{dy}{dx}$.

[6 marks]

(c) Using partial fraction decomposition, find $\int \dfrac{2}{x^2 - 4}\, dx$.

[6 marks]

(d) Find the absolute maximum and absolute minimum points of the function

$$f(x) = (x^2 + 2x)^2 - 1, \quad -1 \le x \le 1.$$

[7 marks]

Continued...

Question 2

(a) Given $f = x^2 + y^2 + 2xy$, where $x = 2s + t$ and $y = s + 2t$. Prove that

$$\frac{\partial^2 f}{\partial s^2} = \frac{\partial^2 f}{\partial t^2}.$$

[6 marks]

(b) The volume, V, of a cylinder with hemispherical ends is given by

$$V = \pi r^2 h + \frac{4}{3}\pi r^3,$$

where r is the radius and h is the height of the cylinder. Suppose the radius of the cylinder is increased by 0.01m from r = 2m and the height is increased by 0.05m from h = 3m. Use total differential to approximate the percentage change in the volume of the cylinder.

[7 marks]

(c) By using the method of Lagrange multipliers, find the maximum of $f(x, y, z) = 200x^2yz$ subject to $x^2 + y^2 + z^2 = 4$.

[12 marks]

Question 3

(a) (i) Given the Maclaurin series for $\sin x$ is

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \text{ for all real } x.$$

Find the Maclaurin series for $f(x) = x\cos(2x)$. Then, give the first four terms of the series.

[6 marks]

(ii) Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)!}{3^n} x^n$.

[6 marks]

(b) (i) Let $z = 1 + i\sqrt{3}$. Find the modulus and the principal argument of z. Then sketch the argand diagram and express z in polar form.

[7 marks]

(ii) Find all the three complex roots of the equation $z^3 = 1 + i\sqrt{3}$.

[6 marks]

Continued...

Question 4

(a) Let $g(x)$ and $h(x)$ be two periodic functions, where:

- $g(x)$ has the Fourier series representation $\dfrac{128}{\pi^3}\sum_{k=1}^{\infty}\dfrac{1}{(2k-1)^3}\sin\dfrac{(2k-1)\pi x}{4}$,

  and

- $h(x)$ is the square wave shown in Figure Q4.

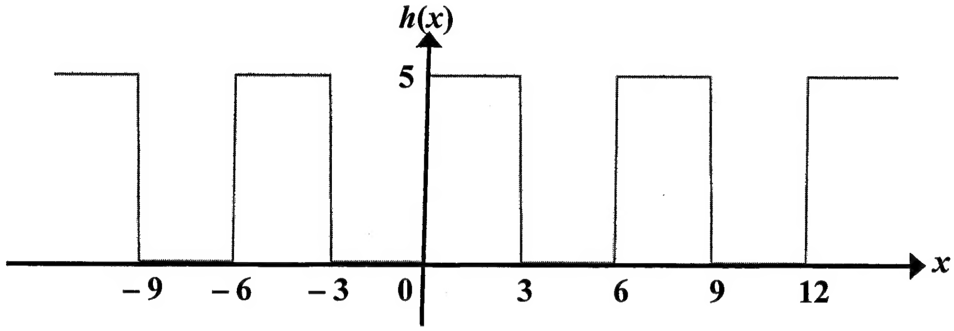

Figure Q4

Answer **any FOUR** of the following:

(i) Determine the period of $g(x)$.

(ii) Is $g(x)$ an even function, odd function, or neither? Justify your answer.

(iii) Determine the period of $g(x)\times h(x)$.

(iv) Would $g(x)\times h(x)$ be odd, even or neither? Justify your answer.

(v) To what value will the Fourier series of $h(x)$ converge at $x=3$?

[8 marks]

(b) A periodic function $f(x)$ of **period 4** is defined over $[-2,2)$ by

$$f(x)=\begin{cases}-3x-2, & -2\le x<0,\\ 3x-2, & 0\le x<2.\end{cases}$$

(i) Sketch the graph of $f(x)$ from $x=-6$ to $x=8$.

[4 marks]

(ii) Develop the Fourier series expansion of $f(x)$.

[13 marks]

End of paper.